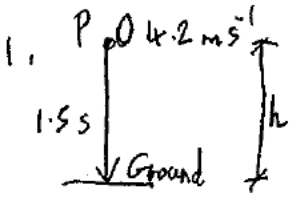
M1 January 2010 Solutions (OCR 4728)

1.



i) $v = u + at$

$v = 4.2 + 9.8 \times 1.5 = \underline{\underline{18.9\ \text{m s}^{-1}}}$

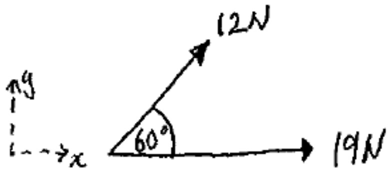
ii) $s = ut + \frac{1}{2}at^2$

$h = 4.2 \times 1.5 + \frac{1}{2} \times 9.8 \times 1.5^2 = \underline{\underline{17.3\ \text{m}}}$

iii) $v^2 = u^2 + 2as$

$v = \sqrt{(4.2^2 + 2 \times 9.8 \times (17.325 - 5))} = \underline{\underline{16.1\ \text{m s}^{-1}}}$

2.

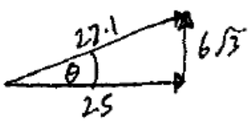


i) Using the vector sum of forces:-

$$R = \begin{pmatrix} 19 \\ 0 \end{pmatrix} + \begin{pmatrix} 12\cos 60° \\ 12 \sin 60° \end{pmatrix} = \begin{pmatrix} 19 + 12/2 \\ \frac{12\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} 25 \\ 6\sqrt{3} \end{pmatrix}$$

$R = \sqrt{(25^2 + (6\sqrt{3})^2)} = \sqrt{(625 + 108)} = \underline{\underline{27.1\ \text{N}}}$

ii)



$\tan\theta = \frac{6\sqrt{3}}{25} \Rightarrow \theta = \tan^{-1}\left(\frac{6\sqrt{3}}{25}\right) = 22.57°$

$\therefore$ The angle between resultant and 12N force $= 60 - 22.57 = \underline{\underline{37.4°}}$

3. i) $\overrightarrow{9\ \text{m s}^{-1}}$ P ● m kg, $\overrightarrow{2\ \text{m s}^{-1}}$ Q ● 0.8 kg $\xrightarrow{\text{collision}}$ $\overleftarrow{3.5\ \text{m s}^{-1}}$ P ● m kg, $\overrightarrow{3.5\ \text{m s}^{-1}}$ Q ● 0.8 kg

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

a) $9m + 2 \times 0.8 = -3.5m + 3.5 \times 0.8$

$12.5m = 1.5 \times 0.8$

$m = \underline{\underline{0.096\ \text{kg}}}$

b) Change in momentum of $P = 9 \times 0.096 - (-3.5 \times 0.096) = \underline{\underline{1.2\ \text{kg m s}^{-1}}}$

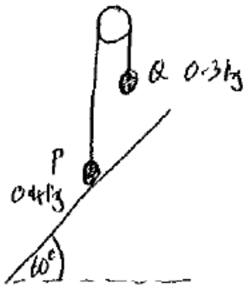
ii) $\overrightarrow{3.5\ \text{m s}^{-1}}$ Q ● 0.8 kg, $\overrightarrow{2.75\ \text{m s}^{-1}}$ R ● 0.4 kg $\xrightarrow{\text{collision}}$ $\overrightarrow{v}$ QR ●● $(0.8 + 0.4)$ kg

$3.5 \times 0.8 + 2.75 \times 0.4 = (0.8 + 0.4)v$

$v = \underline{\underline{3.25\ \text{m s}^{-1}}}$

M1 January 2010 Solutions (OCR 472

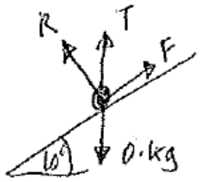
4.



AT Q:-

Resolve vertically:-
$T = 0.3g$ — ①

AT P:-



Resolve ⊥ slope:-
$T\cos 60 + R = 0.4g\cos 60°$ — ②

Resolve // slope:-
$T\sin 60 + F = 0.4g\sin 60°$ — ③

i) a) Substitute ① in ②:- $0.3g\cos 60° + R = 0.4g\cos 60°$

$\underline{\underline{R = 0.1g\cos 60°}}$ (Component ⊥ to plane)

Substitute ① in ③:- $0.3g\sin 60° + F = 0.4g\sin 60°$

$\underline{\underline{F = 0.1g\sin 60°}}$ (Component // to plane)

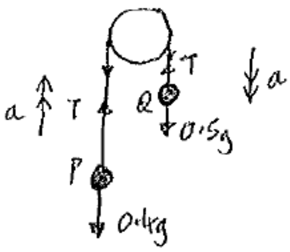
b) As the particle P is in limiting equilibrium, $F = F_{max} = \mu R$

$\therefore \mu R = 0.1g\sin 60°$

$\Rightarrow \mu(0.1g\cos 60°) = 0.1g\sin 60$

$\mu = \dfrac{0.1g\sin 60}{0.1g\cos 60} = \dfrac{\sqrt{3}\times 2}{2} = \underline{\underline{\sqrt{3}}}$

ii)



Resolve vertically at each particle

At P:- $T - 0.4g = 0.4a$ — ①

At Q:- $0.5g - T = 0.5a$ — ②

For acceleration, a, ① in ②:-

$0.5g - (0.4a + 0.4g) = 0.5a$

$0.5g - 0.4a - 0.4g = 0.5a$

$0.1g = 0.9a$

$a = \dfrac{0.1g}{0.9} = \underline{\underline{1.09\ ms^{-2}}}$

Using ①:- $T = 0.4a + 0.4g$

$T = \dfrac{0.4\times 0.1g}{0.9} + 0.4g = 0.4g\left(\dfrac{0.1}{0.9}+1\right) = \underline{\underline{4.36\,N}}$

M1 January 2010 Solutions (OCR 4728)

5. i) To find the acceleration of B in first 20 seconds.

$$v = u + at \Rightarrow a = \frac{v-u}{t} = \frac{11-3}{20} = \underline{\underline{0.4\ ms^{-2}}}$$

When the speed of B is $8\ ms^{-1}$:-

$$v = u + at \Rightarrow t = \frac{v-u}{a} = \frac{8-3}{0.4} = \underline{\underline{12.5\ s}}$$

ii) Distance travelled by A in first 20s $= 8 \times 20 = 160$ m

Distance travelled by B in first 20 s $= \frac{1}{2}(u+v)t = \frac{1}{2}(3+11) \times 20 = 140$ m

When B overtakes A:-

Distance travelled by A $= 8t$

Distance travelled by B $= 140 + 11(t-20)$

As distances are the same, $8t = 160 + 11(t-20)$

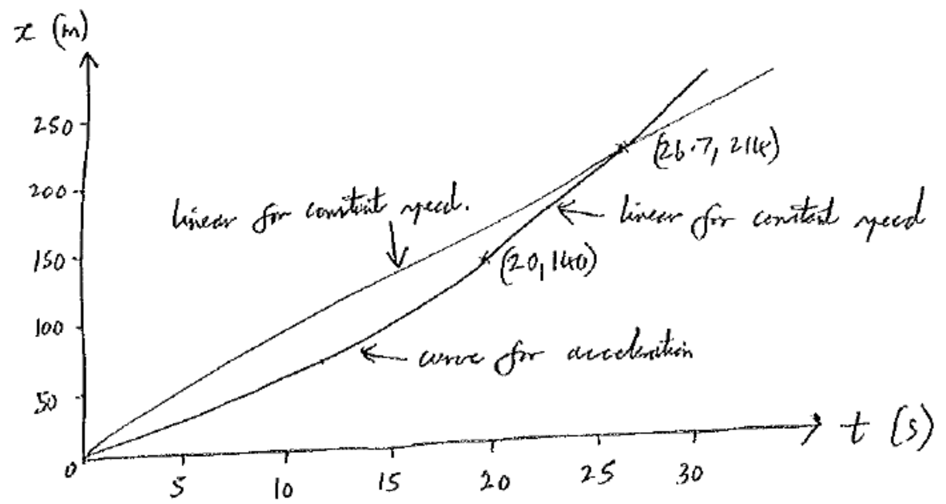
$$8t = 160 + 11t - 220$$

$$220 - 160 = 11t - 8t$$

$$80 = 3t$$

$$\underline{\underline{t = 26.7\ s}}$$

iii) Distance to when B overtakes A $= 8 \times 26.7 = 214$ m



M1 January 2010 Solutions (OCR 4728)

6. i) $v = 0.006t^2 - 0.18t + k$

Acceleration $= \frac{dv}{dt} = 2 \times 0.006t - 0.18 = \underline{\underline{0.012t - 0.18}}$

ii) When the speed is at a minimum, acceleration is zero

$\therefore\ 0.012t - 0.18 = 0$

$t = \frac{0.18}{0.012} = 15s$

When $t = 15s$, $v = 0.65 ms^{-1}$ $\therefore\ 0.65 = 0.006 \times 15^2 - 0.18 \times 15 + k$

$k = 0.65 - 0.006 \times 15^2 + 0.18 \times 15 = \underline{\underline{2}}$

iii) $v = \frac{dx}{dt} = 0.006t^2 - 0.18t + 2$

$x = \int (0.006t^2 - 0.18t + 2)\, dt$

$x = 0.002t^3 - 0.09t^2 + 2t + c$

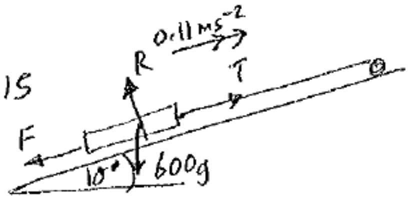
When $t = 0, x = 0 \Rightarrow c = 0$

$\therefore\ x = 0.002t^3 - 0.09t^2 + 2t$

Swimmer reaches end of pool in 28.4s

Length of the pool $= 0.002 \times (28.4)^3 - (0.09 \times (28.4)^2) + 2 \times 28.4 = \underline{\underline{30m}}$

7. $\mu = 0.15$



Resolve ⊥ slope :-

$R = 600g \cos 10°$

Resolve // slope :-

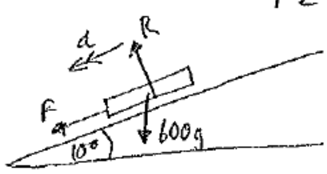
$T - F - 600g \sin 10° = 600 \times 0.11$ —①

i) As the log is moving, $F = F_{max} = \mu R = 0.15 \times 600g \cos 10°$ —②

Substitute ② in ① :- $T - 0.15 \times 600g \cos 10° - 600g \sin 10° = 600 \times 0.11$

$T = 600 \times 0.11 + 0.15 \times 600 \times 9.8 \cos 10° + 600 \times 9.8 \sin 10° = \underline{\underline{1960N}}$

ii) a)



Resolve ⊥ slope :- $R = 600g \cos 10°$

Resolve // slope :- $F + 600g \sin 10° = 600a$ —①

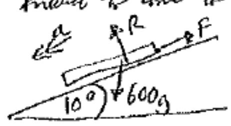
As the log is moving, $F = F_{max} = \mu R = 0.15 \times 600g \cos 10°$ —②

Substitute ② in ① :- $(0.15 \times 600 \times 9.8 \cos 10° + 600 \times 9.8 \times \sin 10°)/600 = a = \underline{\underline{3.15 ms^{-2} \text{ deceleration}}}$

b) Velocity when cable breaks :- $v^2 = u^2 + 2as \Rightarrow v = \sqrt{0^2 + 2 \times 0.11 \times 10} = 1.48 ms^{-1}$

Time to come to rest after cable breaks :- $v = u + at \Rightarrow t = \frac{v-u}{a} = \frac{0 - 1.48}{-3.15} = 0.47s$

Distance to come to rest after cable breaks :- $v^2 = u^2 + 2as \Rightarrow s = \frac{v^2 - u^2}{2a} = \frac{0 - (1.48)^2}{2 \times 3.15} = 0.348m$



Acceleration down slope $= (600 \times 9.8 \sin 10° - 0.15 \times 600 \times 9.8 \times \cos 10°)/600 = 0.254 ms^{-2}$

Time to travel 10.348m down slope :- $s = ut + \frac{1}{2}at^2 \Rightarrow 10.348 = 0 + \frac{1}{2} \times 0.254 t^2 \Rightarrow t = \sqrt{\frac{10.348 \times 2}{0.254}} = 9.03s$

Time from cable breaking $= 0.47 + 9.03 = \underline{\underline{9.5s}}$